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DISCUSSION ON "THE EFFECT OF IRON IN DISTORTING ALTERNATING-CURRENT WAVE-FORM" AT NEW YORK, SEPTEMBER 28, 1906.

**Charles Proteus Steinmetz:** This paper deals with the wave-shape distortion produced in alternating-current circuits by the introduction of iron. It is a theoretical paper, and while of scientific interest appears at first of rather little practical value to the electrical engineer. There is, however, to-day only a very short step between pure scientific investigation and engineering practice; and I hope to show you that the phenomena dealt with in this paper, and similar phenomena, are of very great practical importance in alternating-current dis-

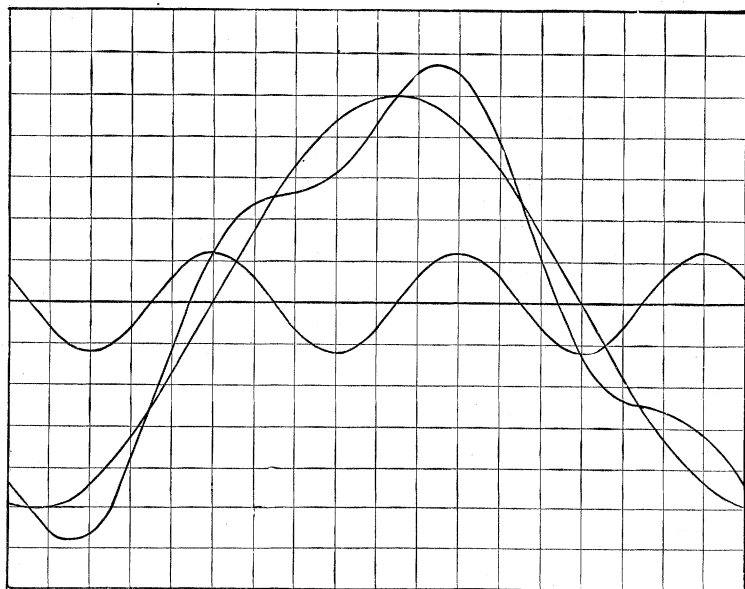


FIG 1

tribution; that is, wave-shape distortion may lead to effects not only very marked and pronounced but occasionally disastrous. In general, in investigating the effect of iron in alternating-current circuits, the curve of exciting current is calculated from the hysteresis cycle of the iron. Dr. Bedell proceeds inversely by superposing different harmonics of current. From these complex currents he produces a hysteresis loop, noting whether this hysteresis loop is a reasonable one or not, and deriving therefrom relations regarding the relative intensity and phase of the triple harmonic in the wave of exciting current. As far as the investigation goes, it extends only to the fundamental and triple harmonics; the investigation of higher harmonics is left to a future occasion.

These higher harmonics obviously modify to a certain extent the conclusions arrived at by assuming merely the fundamental and triple harmonic as present. For instance, by superposing a triple harmonic upon the fundamental wave, one gets a wave of the shape shown in Fig. 1., with a hump on the rising side and a hollow on the decreasing side. Introducing a triple harmonic of higher amplitude causes the hump to develop into a double peak as in Fig. 2. It is obvious that a double peak cannot exist, because whatever relation may exist between the magnetism and the magnetizing current, the current must rise as long as the magnetism rises; and therefore the maximum possible value of the triple harmonic is that value which does not yet give a

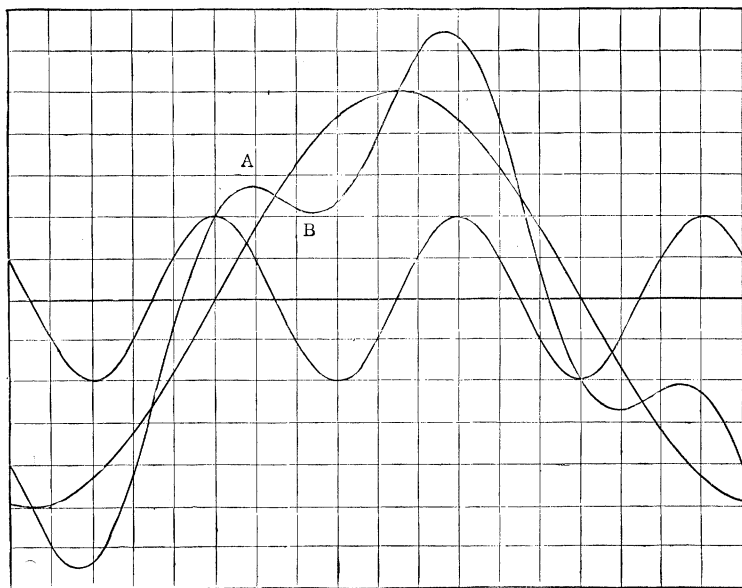


FIG 2

downward bend, but merely flattens the current wave on the rising side. This maximum amplitude of the third harmonic can, however, be exceeded if higher harmonics are present. Assume for instance a fifth harmonic which has such a phase relation as to be negative at A Fig. 2, and positive at B, and then superpose this fifth harmonic on the double-peaked wave; it will be seen that it cuts off the peak and fills up the hollow, and gives a wave which represents a possible hysteresis cycle, as seen in Fig. 3. The effect of the fifth harmonic, then, is to permit the existence of a triple harmonic, larger than could exist in the absence of the fifth harmonic. It is quite probable that not infrequently in the exciting current there occur triple-harmonic currents higher than the maximum value cal-

culated in Dr. Bedell's paper, and the double peak is cut off by the fifth harmonic. The fifth harmonic being in phase, approximately, at the maximum value of magnetism, is approximately in opposition at the zero of magnetism, where the double peak tends to form. This brings up the question of the desirability of extending Dr. Bedell's investigation to still higher harmonics, the fifth, seventh, ninth, etc.

An interesting investigation of the wave-shape distortion of the exciting current is given in a paper presented to the Institute May 1896 by C. K. Huguet. It was this: let there be a sine wave of electromotive force, producing a sine wave of magnetism,

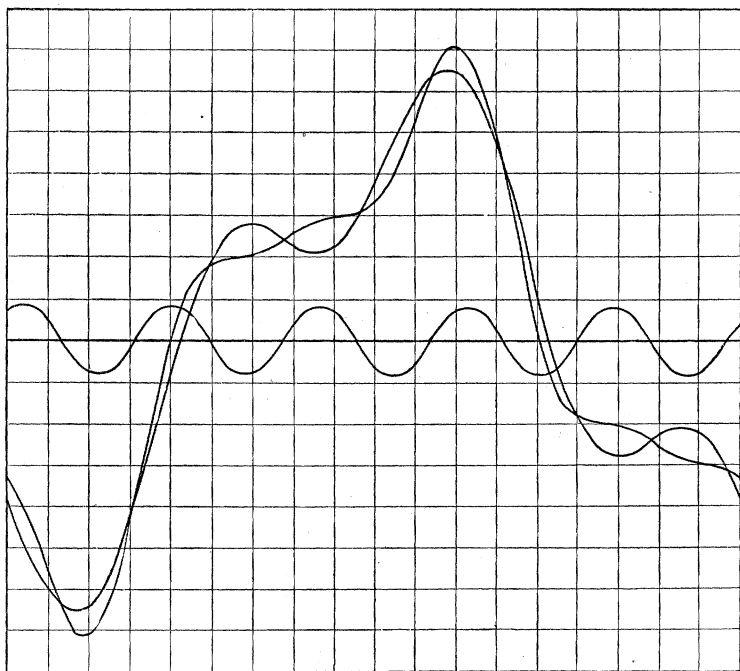


FIG 3

and from the hysteresis cycle construct the wave of exciting current. This exciting current can be resolved into two components: one component symmetrical with regard to the wave of magnetism, or wattless current; the other symmetrical with regard to the wave of electromotive force, representing power. The component in phase with the magnetism will be found to be greatly distorted, while the component in phase with the electromotive force is practically a sine wave, as shown in Fig. 4. I have checked this in quite a number of cases and it agrees nicely, except that there always are some small very high harmonics in the energy wave which makes this curve horizontal at the

zero value. That is, the harmonics symmetrical with regard to the electromotive force are noticeable only at the zero point, as a flattening out. The magnetism curve at this point is horizontal, so that the resultant current curve must be horizontal also. This could be expressed by stating that the distortion of the wave of the exciting current is due, not to the energy lost in the iron, but to the magnetic characteristic or the bending of the saturation curve, and therefore it is this curve which we should endeavor to construct, the magnetic characteristic as it would be given by a magnetic cycle, in the absence of hysteresis loss. This would probably give approximately the higher harmonics in the exciting curve wave.

Sometime in 1881 or 1882 Dr. Froehlich noticed that the magnetic characteristic of the dynamo machine could be ap-

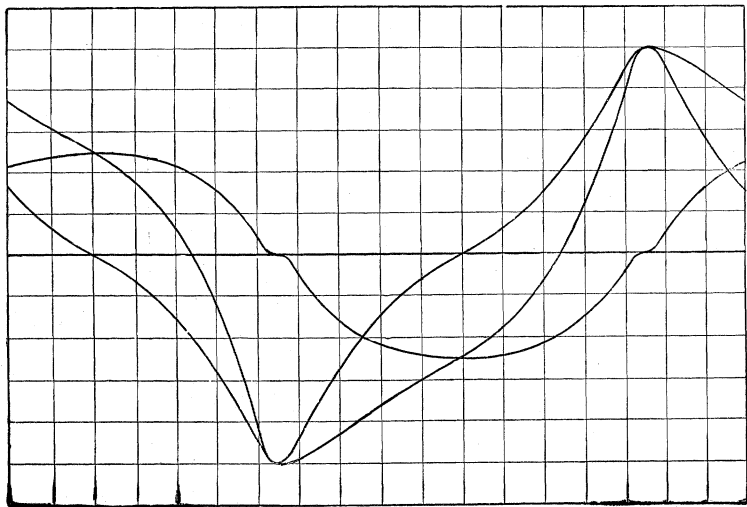


FIG 4

proximately represented by a parabolic curve. Dr. Kennelly showed, in 1891\*, that the  $BH$  curve, or magnetic characteristic of iron, for the higher values, could be expressed by a parabolic curve, an equation of the second degree. Using this equation of a parabola for the relation between  $B$  and  $H$ , there could be found a strictly mathematical curve, about like  $B_0$  in Fig. 5, which combined with a sine wave representing the hysteresis loss, would fairly closely represent the distorted wave of exciting current.

In dealing with hysteresis we have to keep in mind the difference between magnetic hysteresis and the energy lost in the iron. If iron is exposed to an alternating magnetic field, the loss of energy that takes place in the iron, by some form of magnetic

\*Magnetic Reluctance, by A. E. Kennelly, TRANSACTIONS A. I. E. E. Vol. 8, page 485.

friction, is usually expressed as "molecular magnetic friction." This loss seems to be constant, independent of the frequency or wave-shape, depending only on the maximum values of the magnetic induction. If the alternating electrical circuit is the only source of power, and no power is consumed outside of the iron, then the power consumed by molecular magnetic friction must be supplied by the alternating circuit, and is supplied in the form of a hysteresis cycle. In this case molecular magnetic friction and magnetic hysteresis coincide, or rather the magnetic hysteresis measures the molecular magnetic friction. As soon, however, as there is another source of power present, or power can be consumed elsewhere, this coincidence disappears and there is no inherent relation between molecular magnetic friction and magnetic hysteresis. This was shown first by the experiments of Gerosa and Finzi 1891, recorded by Ewing in his work

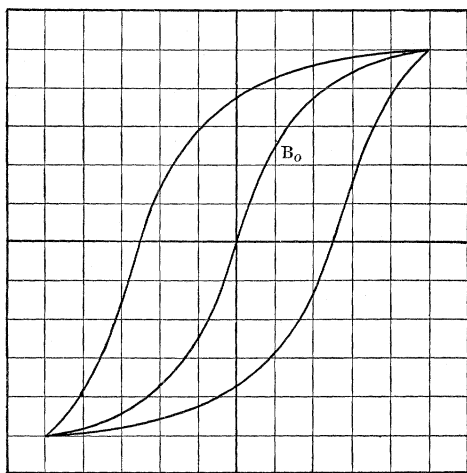
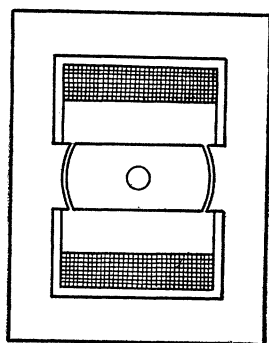


FIG. 5.

on magnetism. If an alternating current is sent through the magnetic circuit parallel to the lines of magnetic force, at a frequency which is high compared with the frequency of the magnetic cycle, then the hysteresis loop more or less completely collapses; but the molecular magnetic friction still remains, only that now the longitudinal alternating current supplies all or nearly all the power. The reverse is the case where there are loose laminations in a transformer. It will be found that the hysteresis loop is extended and the electric circuit in the form of a hysteresis loop supplying more power than is consumed in the iron by molecular magnetic friction; the difference is consumed in the vibration of the laminations, resulting in noise. Where energy is supplied from an outside source, it may go so far as not only to make the hysteresis loop disappear, but to make it

negative. Some interesting conditions where the hysteresis loop could be flattened out or turned over were investigated by Mr. Eickemeyer and myself in 1891, on a magnetic circuit of the shape of that of a shell-type transformer, shown in Fig. 6, in which the central core could be rotated. We found that such an arrangement when running at synchronism would give all kinds of hysteresis loops; for instance, that the more the apparatus as motor was loaded the fatter became the hysteresis loop. Whenever the friction is supplied by an outside source the hysteresis loop collapses, and reverses by driving the rotor by power. Some hysteresis loops of this apparatus are given in my second paper on the Law of Hysteresis.\*

These overturned magnetic cycles differed considerably from the typical hysteresis cycle, Fig. 5. A typical hysteresis cycle,



*Diagrammatic View of  
Eickemeyer Humming Bird*

FIG. 6.

however, can be made to contract, disappear, and reverse in the following manner:

Two equal exciting coils  $A$  and  $B$ , in Fig. 7, at right angles with each other in space, are energized by two equal sinusoidal quarter phase e.m.f.'s. so giving a uniformly rotating magnetic field. In the center of this field is a movable iron disc,  $C$ . With this disc at standstill, the line of resultant magnetism in the disc  $Y_1 Y_1^1$ , lags behind the line of resultant rotating m.m.f.  $XX^1$  of the exciting coils, by the angle of hysteretic lead  $\alpha$ , and the relation of impressed e.m.f. and so of magnetic flux, and of exciting current in the coils  $A$  and  $B$  gives the typical hysteresis cycle, Curve I, Fig. 8.

With the disc  $C$  rotating below synchronism, the angle  $XOY = \alpha$  remains the same, the hysteresis cycle, and thereby the power consumed in the exciting coils, is the same; but the molecular

\*TRANSACTIONS, A. I. E. E., 1892, vol. 9, p. 3.

magnetic friction in the disc, while the same per cycle, decreases with increasing speed, proportional to the decreasing frequency of slip. The difference in the power consumed by hysteresis in the exciting coils, and the power consumed by molecular magnetic friction in the disc, is converted into mechanical work, and such an apparatus, which I called "hysteresis motor," so gives constant torque at all speeds, up to synchronism. If this torque is more than the friction torque, the disc accelerates up to synchronism. At synchronism, molecular magnetic friction disappears, and the line of resultant magnetic flux retains a constant position with regard to the iron, and all the power given by the exciting currents in the form of the hysteresis loop is converted into mechanical power. If this is more than the power consumed by mechanical friction, the line of magnetization runs ahead by the acceleration of the disc. to  $Y_2 Y_2^1$ , the angle of hysteretic advance  $X O Y$  decreases, and the hysteresis cycle of

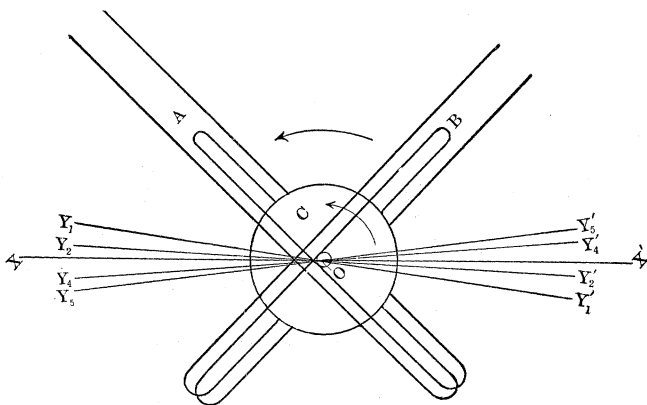


FIG. 7.

the exciting coils so contracts, to Curve II, Fig. 8, giving an area corresponding to the friction torque only. If now the friction torque is supplied by a mechanical driving force, and the disc  $C$  not called upon to do any mechanical work, it runs ahead until its line of magnetization  $Y Y^1$  coincides with the line of resultant m.m.f.  $X X^1$ ; that is, the hysteretic angle  $\alpha$  disappears, and the curve of magnetism is symmetrical with the curve of exciting current, or the hysteresis loop collapses to Curve III, Fig. 8.

Still greater driving force impressed upon the disc  $C$ , sends the line of resultant magnetization ahead of  $X X^1$ , to  $Y_4 Y_4^1$ , the angle of hysteretic advance  $\alpha$  becomes negative, and the hysteresis loop opens up again, to Curve IV, Fig. 5, but is traversed now in opposite direction, or overturned, representing production of electric power. In this case, the curve of exciting current in  $A$  or  $B$  has the reverse shape; a hollow on the rising, a hump on the decreasing side.



With increasing driving power, the overturned hysteresis loop IV fattens, until it reaches the same shape as I, but traversed oppositely, and then synchronism is broken, and disc *C* speeds up. Above synchronism, the hysteresis cycle has the normal shape I, but is overturned, the angle of hysteretic advance of phase has reversed its sign, and molecular magnetic friction again consumes power in the disc; but this power is now given by the mechanical driving power, and not by the electric circuit.

Below synchronism, a constant amount of electric power is consumed; above synchronism, a constant amount of electric power is generated in the exciting coils, irrespective of the speed, while the power consumed by molecular magnetic friction in the

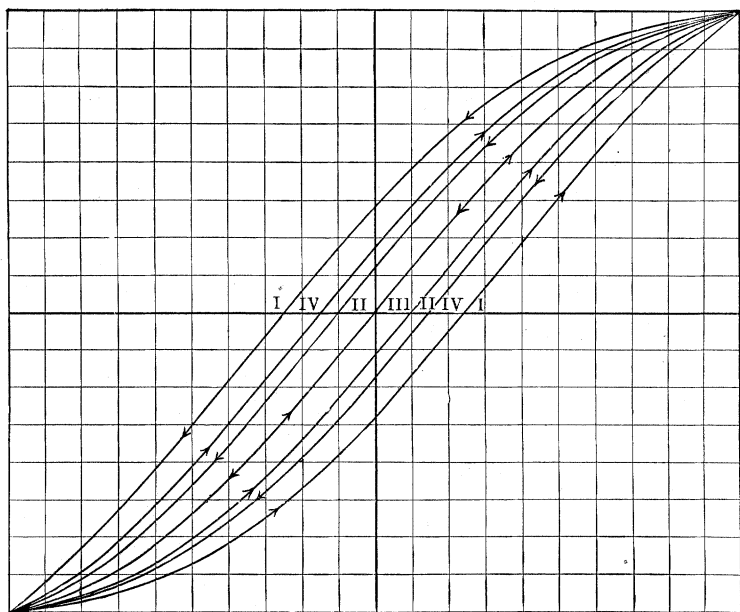


FIG. 8.

disc varies proportional to the slip from synchronism, but is the same above as below synchronism.

The bearing of these wave-shape phenomena on practical engineering will now be considered.

If there be a sine wave of impressed electromotive force, *E*, Fig. 9, or rather of counter electromotive force, it produces a sine wave of magnetic flux *B*. This sine wave of magnetic flux causes an exciting current to flow which is distorted by hysteresis, or rather, as we may say, by the magnetic characteristic, and is given by Curve I. If, however, the transformer is traversed by a sine wave of exciting current, *I* in Fig. 10, we get by the hysteresis loop a wave of magnetism, which is not a sine wave, but which

is hollow on the rising side, rises very rapidly and decreases very slowly, at first, and then very rapidly. That is, the wave of magnetism has a pronounced flat top, and the wave of e.m.f. induced thereby is very low for a considerable part of the period, then rises very sharply to a high triangular peak, and falls off just as rapidly, as shown by  $E$  in Fig. 10. This peak rises to nearly twice the maximum value of the fundamental sine wave,  $E_1$  Fig. 10. That is, with a sine wave of current traversing an ironclad magnetic circuit, the e.m.f. wave is greatly distorted, and the magnetic circuit generates higher harmonics of e.m.f. mainly of triple frequency.\*

Very interesting phenomena result from this wave-shape dis-

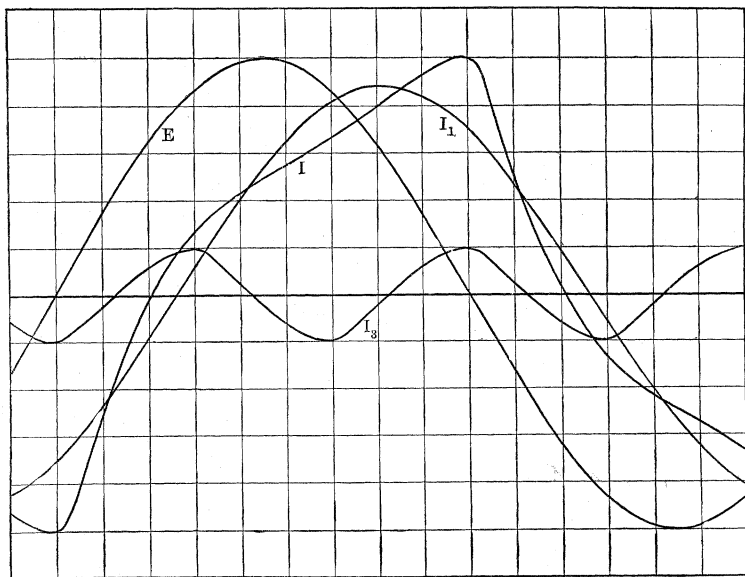


FIG. 9.

tortion by the magnetic cycle, if transformers are grouped in such a manner that certain harmonics can not develop.

In a three-phase system with three transformers connected in delta or ring connection, and a sine wave of impressed e.m.f., the exciting current in the transformers has the usual shape,  $I$  in Fig. 9, containing a pronounced third harmonic, which is shown separately as  $I_3$  in Fig. 9, together with all its higher harmonics or

\*For instance with the hysteresis cycle Fig. 5, and a current  $I = 10 \sin(\phi + 30^\circ)$ , the e.m.f. is approximated by the equation:  $E = 11.67 \cos(\phi + 2.5^\circ) + 6.64 \cos(3\phi - 3.4^\circ) + 3.24 \cos(5\phi - 11.9^\circ) + 1.8 \cos(7\phi - 10.7^\circ) + 1.16 \cos(9\phi - 4.5^\circ) + 0.80 \cos(11\phi - 22^\circ) + 0.53 \cos(13\phi - 26^\circ) + 0.19 \cos(15\phi - 15^\circ) + \dots$

"overtunes." The current in the three-phase lines can not contain any third harmonic: the current in line 1 is the resultant of the currents flowing from line 1 to 2, and from 1 to 3, and since these two currents are 60 degrees apart in phase, their third harmonics are 180 degrees apart, or in opposition, hence cancel. That is, the triple-harmonic component of the exciting current circulates in a local circuit through the transformer triangle, without reaching the three-phase lines. All the other harmonics of exciting current appear in the line current.

If the primary coils of the transformers are connected in *Y* or star connection, the secondaries in delta, the primary exciting current does not contain any third harmonic, but the triple

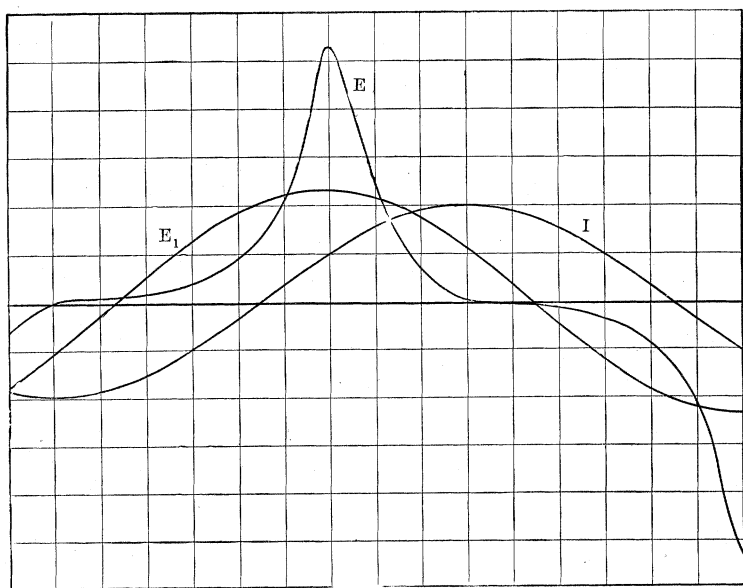


FIG. 10

harmonic of excitation circulates in the secondary transformer triangle in local circuit.

Perhaps still more interesting is the case of three transformers, connected with their primaries and secondaries in *Y* or star connection in a three-phase system with sinusoidal e.m.f. impressed upon the lines.

In a three-phase system, the three e.m.f.'s. from the lines to the neutral are 120 degrees apart and so are the three currents. With a sine wave of impressed e.m.f., if the e.m.f.'s. between lines and neutral were sine waves also, the three exciting currents would contain strong third harmonics. Since these currents are 120 degrees apart, their third harmonics would be  $3 \times 120 = 360$  degrees apart, or in phase; that is, all three flow simultaneously toward the neutral. If now the neu-

tral is isolated these triple-frequency components of exciting current have no circuit; that is, cannot flow, and the e.m.f.'s. between lines and neutral therefore can not be sine waves, but must be distorted by the suppression of the triple-frequency component of exciting currents. This distortion of e.m.f. wave can be due only to a third harmonic and its overtones, which cancel by combining two such e.m.f.'s. between line and neutral, under 60 degrees to the impressed e.m.f. while all the other harmonics would not cancel, but appear in the impressed e.m.f. which was assumed as a sine wave.

It follows herefrom, that with a sine wave of three-phase e.m.f. impressed upon a system of Y-connected transformers with isolated neutral, the e.m.f.'s. between lines and neutral, or potential differences at the transformer terminals, cannot be sine waves, but contain a pronounced third harmonic and its overtones, but no other harmonics; while the exciting currents contain no third harmonic or multiple thereof, but all other harmonics.

For the hysteresis cycle, Fig. 5, and a sine wave of impressed e.m.f., Fig. 11, shows the wave of exciting current  $I$ , the transformer e.m.f. or voltage between line and neutral,  $E$ , its fundamental sine wave,  $E_1$ , and the sum of all its harmonics,  $E_3$ . As seen, the e.m.f.,  $E$ , is peaked, while the wave of magnetism (not shown) has a flat top. The triple-harmonic e.m.f.,  $E_3$ , is nearly half the fundamental,  $E_1$ , in this case. From this peaked wave  $E$  may result an increased insulation strain, but a decreased hysteresis loss. The e.m.f. on the transformer is higher than the line e.m.f. divided by  $\sqrt{3}$ .

In cases where the neutral is not grounded, these harmonics of electromotive force appear as potential difference between neutral and ground. The neutral of Y-connected three-phase transformers therefore is not at ground potential, but may have a considerable potential difference against ground, of triple frequency: the third harmonic,  $E_3$  Fig. 11, which is generated by the magnetic cycle of the transformer.

With a grounded neutral; that is, zero potential difference between neutral and ground, but no other ground on the system, the triple harmonic of exciting current still cannot flow, and the potential difference in the three transformers still contains a third harmonic. Since all these triple harmonics are in phase with one another, it means that all three lines rise and fall simultaneously, or in synchronism with one another against ground, or a triple-frequency voltage appears between the three lines of the three-phase system and the ground, which may have a fairly considerable magnitude as seen in Fig. 11. Suppose now these transformers with grounded neutral feed into a long distance transmission system. We have a circuit from the grounded neutral, over the inductance of the three transformers in multiple, and back to ground over the capacity of the three transmission lines against ground, with a triple-frequency im-

pressed e.m.f., the third harmonic generated in the transformers. There is a high frequency e.m.f., in series with inductance and capacity. Such a combination may, under unfavorable conditions, be serious in originating surges in the system, against ground, of more or less destructive voltage. But even if no serious high potential phenomena occur, the rise and fall of the whole system at triple frequency may give electrostatic induction on neighboring circuits, as telephone lines, etc. Suppose we ground the neutral

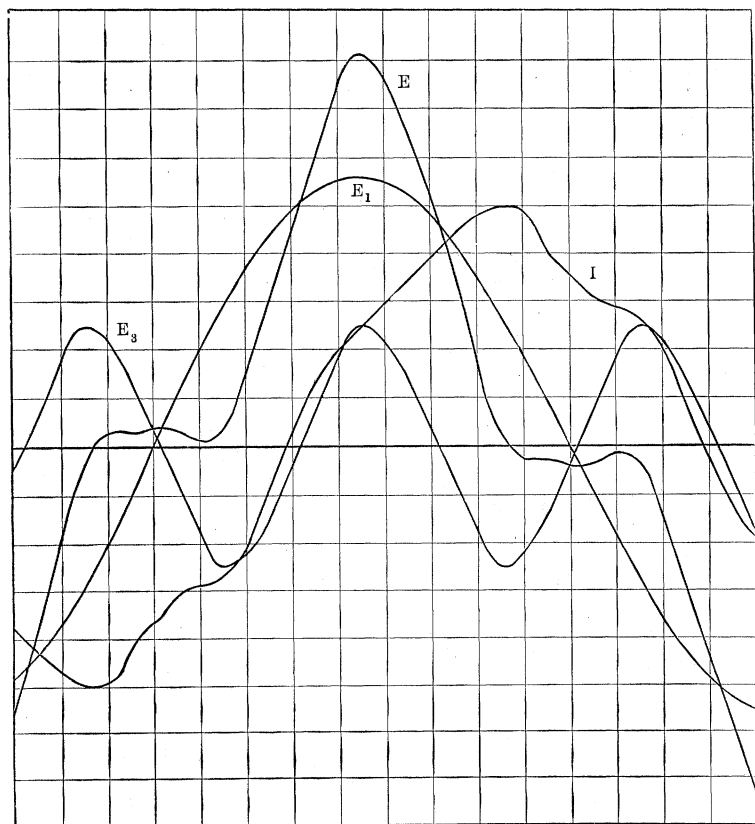


FIG. 11.

of the step-down transformers also, and connect their secondaries in delta. Then the triple-frequency electromotive force disappears and the triple-frequency current flows over the ground and circulates in the secondary delta of the step-down transformers. We have then in the system a triple-frequency current which flows over all three lines in parallel and back over the ground; and while triple-frequency electrostatic induction disappears there appears electrodynamic induction.

Similar phenomena also occur with alternating-current generators. In a three-phase generator with the three coils Y-connected, if there is a triple-frequency electromotive force in each phase, a potential difference of triple frequency exists between neutral and ground, or, with grounded neutral, between the three lines and ground. These electromotive forces are in series short-circuited upon themselves, in the three-phase delta-connected generator. There is, however, a very essential difference between this case and the corresponding case of the transformer. In the case of transformers, we can only get the triple-frequency component of the exciting current; that is, the current which can flow between neutral and ground, or circulate locally in the delta, is limited. With a generator, it is a short-circuit current of the induced electromotive force of triple frequency. Such currents circulating in the windings of delta-connected generators were observed years ago. In many cases they may have been attributed to abnormally great hysteresis losses, and escaped attention. In the Y-connected generator, you may have triple-frequency electromotive forces in the phases which do not appear in the terminal voltage, and give a triple-harmonic e.m.f. from the neutral against ground, and if we get a path for this triple harmonic, we may get currents which in this case are not merely two or three per cent. of the full load current—the triple-frequency component of the transformer exciting current—but may be full-load current or more. If the phase relation of the triple-frequency harmonic with the fundamental is the same in all generators of the system there would be no current in the neutral. If we run two machines at different excitation, one higher and the other lower, then a current flows between the two machines which is a wattless current, magnetizing the under-excited and demagnetizing the over-excited machine. The terminal voltages are not quite in phase with the induced voltage, but in phase with each other, since the machines are connected together. The triple-frequency voltages so give a resultant, and thus a current over the neutral, which may reach very high values. If we have two generators, one having a triple-frequency e.m.f., we get the same phenomenon of a triple-frequency current; but this current is not limited and may occasionally reach values comparatively high, and that is why it is not safe freely to ground the generator neutrals. If the generators are to be grounded, they should be grounded through a resistance limiting the neutral current, or they must have practically the same wave shape and the excitation must be kept practically alike in each generator.

**Philip Torchio:** I wish to ask Mr. Steinmetz about the third harmonic short circuit between three-phase generators Y-connected with the grounded neutral. One of the largest companies in New York tried at the start to operate all the generators engine-driven with the neutral grounded, and they found a large short-circuit current between the neutral of

different generators, evidently due, as explained by Mr. Steinmetz, to the third harmonic. In two other plants operating exclusively with turbine-generator sets, no short-circuit current of the neutral has been manifested by the ammeter, all the neutrals of the Y-connected generators being dead grounded. Will Mr. Steinmetz explain why the engine-driven generators give the third harmonic short-circuit current, while the turbine-driven generators do not give such current?

**Chas. P. Steinmetz:** I think I can explain that. The turbine generators were all alike, running with identical wave shapes at equal excitation. I do not know what station is referred to, but if I guess correctly, in the same station some larger turbine-generator sets were afterward installed, and between the old machines and the new ones, very considerable currents were found over the neutral. The question is, whether the triple harmonics are identical and have the same phase, or whether they are not identical and have not the same phase.

**W. S. Franklin:** It is not very often that any of us gets a chance to find fault with what Dr. Steinmetz says, but there is one thing which he mentioned to-night which I wish to criticize, and that is the idea of a hysteresis loop connected with a rotating disc when the flux remains at one constant value; unless, indeed, Dr. Steinmetz means to compare the magnetizing current in the horizontal coil with the vertical component of the magnetism.

Two or three points now which are chiefly of interest in matters educational. In the first place I want to call attention to the use of the word "sinusoidal." In the study of mechanics the word "harmonic" has come into almost universal use for designating that type of motion which is exemplified in the swinging of a pendulum. That is sinusoidal motion. We should adopt the term harmonic, and speak not of sinusoidal, but of harmonic currents and harmonic electromotive forces.

Another point concerns the assumption which is made in all alternating-current treatises as to the harmonic character of electromotive forces and currents generated by alternators. It seems to me that it is a false idea which many people have gotten into, that this assumption places a limitation on the theory of alternating currents, for this reason: given an alternator which develops an electromotive force of any complicated wave-shape whatever, and let it be required to determine the current produced by the electromotive force. This problem resolves itself into a series of problems, each one of which is an ordinary simple harmonic alternating-current problem. The first thing to be done is to resolve the electromotive force into harmonics, and then treat each harmonic electromotive force by itself, and discuss the currents produced. If you limit yourself to the fundamental, you have only solved one problem of the series, ignoring all the others.

In regard to the matter of the magnetizing current, I will

call attention to one point, and that is, that we have had two meanings attached to the term angle of hysteretic advance; one by Dr. Bedell and one by Dr. Steinmetz. I do not think, however, that it is important that this term should be standard, because we do not use it very much.

In regard to the representation of harmonic electromotive forces and currents in vector diagrams, I wish to call attention to two distinct ideas that are involved. First, there is the idea of representing what actually takes place in a circuit; in this case the rotating vectors represent the successive instantaneous values of current and voltage; that is, they represent the actual physical facts. Secondly, there is the idea of getting geometrical representations of formulas. This second idea seems to be in Dr. Bedell's mind. Thus a given current of fundamental frequency and given current of triple frequency when superposed give an effective current which is equal to the square root of the sum of the squares of the two; therefore Dr. Bedell chooses to represent the triple-frequency current by a line at right angles to the plane of the fundamental diagram. That is all very well, but we must keep in mind that we are using the diagram merely as the picture of a formula and not as a representation of physical actions. For my part I prefer to limit the vector diagram to the representation of physical action and I always use the idea of a rotating vector for representing in the students' mind the successive instantaneous values of current and voltage.

**Frederick Bedell:** Professor Franklin's remarks well accord with our own views. It has been brought out in the paper that at least two definitions may be given to the angle of hysteretic advance, these corresponding to the angles  $\alpha$  and  $\phi$  as used by the authors; Professor Franklin emphasizes this. The relation between  $\alpha$  and  $\phi$  is given in Fig. 16, and our purpose has been to distinguish clearly between them. Professor Franklin will find that his definition is our  $\alpha$ , the value as found by measurement with ammeter, voltmeter, and wattmeter.

Professor Franklin also expresses our views in regard to the significance of the geometrical construction. It is merely a picture which helps us to understand some relations. These relations we get more clearly in the diagram, but the diagram is not in any wise a physical representation of the facts.

**W. S. Franklin** (by letter): Two additional points were touched upon in my discussion, namely, (a) in connection with Dr. Steinmetz's reference to Froelich's equation to the  $B$  and  $H$  curve I stated it as my opinion that there are certain physical relations which are essentially irrational and erratic, that such relations can never be formulated in the sense in which Kepler formulated the relations involved in planetary motion, and that we ought to give up an idea which seems to be quite prevalent—the idea that way back somewhere in the region of ideality, wherever that may be, there is a formula that will reduce any



physical relation to a rational basis. I discuss this point rather fully on page 285 of Vol. XX of the Institute TRANSACTIONS.

(b) In connection with Dr. Steinmetz's reference to the influence of vibration on the  $B$  and  $H$  curve, I called attention to a paper of mine\* in which vibrations were used in the determination of what I call a normal curve of  $B$  and  $H$ .

**C. P. Steinmetz** (by letter): (1) Regarding the angle of hysteretic advance  $\alpha$ , I always define this as the phase-angle between the equivalent sine wave of exciting current and the equivalent sine wave of electromotive force induced thereby; that is, the angle given by ammeter, voltmeter, and wattmeter reading, after correcting for the  $I^2R$  (which is best done by using an exploring coil for the potential circuit of wattmeter and voltmeter). In the hysteresis motor referred to in the discussion, it can be shown that the space-angle between the resultant magnetic flux and the magnetomotive force equals the angle of hysteretic advance of phase of the exciting coils, and this space-angle was therefore referred to as the hysteresis angle.

(2) In general, I fully agree with Professor Franklin regarding empirical equations. I do not consider the parabolic law of magnetic induction:

$$B-H = \frac{H}{\alpha + bH}$$

as an empirical law, however, but rather as a rational equation approximating the  $B-H$  curve, and the deviations of the induction curve from this equation as due to secondary phenomena not included in the equation: molecular magnetic friction, which causes a deviation, especially at lower densities; lack of homogeneity, especially noticeable at intermediate inductions (most pronounced in cast iron), etc.

If I remember correctly, Fröhlich proposed this equation, not from experimental data—which at his time were hardly sufficient—but by the following reasoning: magnetic induction in iron, etc., reaches an absolute limiting, or saturation value. This has been proved by Ewing for the ‘metallic magnetic induction’  $B-H$ . When approaching magnetic saturation, the magnetic permeability must therefore decrease, and it is reasonable to assume that the permeability is proportional to the remaining magnetizability of the iron, or to the difference of the induction  $B$  from its saturation value  $S$ . This gives Fröhlich's equation:

$$\mu = \alpha (S - B)$$

or, since  $\mu = \frac{B}{H}$ :

$$\frac{B}{H} = \alpha (S - B)$$

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\**Physical Review*, Vol. VIII, pages 304–309.

$$B = \frac{H}{\frac{1}{\alpha S} + \frac{1}{S}} = \frac{H}{A + B H}$$

or, substituting the reluctance:

$$\rho = \frac{1}{\mu} = \frac{H}{B}$$

it gives Kennelly's equation:

$$\rho = \frac{1}{\alpha S} + \frac{H}{S} = C + D H.$$

These equations obviously do not apply to the total induction  $B$ , but to  $B_0 = B - H$ : the difference, however, becomes noticeable only at very high magnetomotive forces

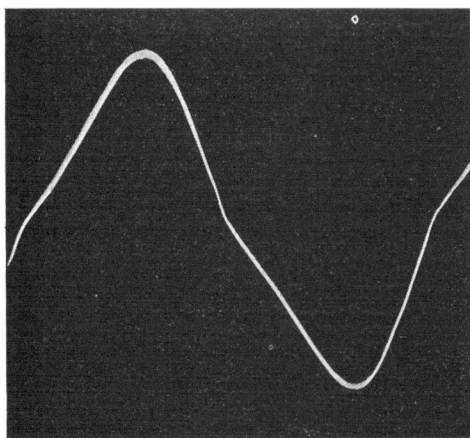


FIG. 1.

**Harold Pender** (by letter): The effect of iron in distorting the wave-form of current and pressure is of particular importance in determining the energy-loss in small samples of iron by the wattmeter method. In this method of measurement the maximum flux density is usually calculated from the effective pressure measured by a voltmeter across the terminals of the magnetizing winding on the sample; or the induced pressure in a secondary winding on the sample may be measured, this method eliminating the resistance-drop in the magnetizing winding.

In general the maximum flux density is proportional to the average value of the induced electromotive force over half a cycle; only in case of a sine wave of induced electromotive force

is the flux density proportional to the effective value. Could a sine wave of electromotive force be impressed directly on a magnetizing winding without resistance, the counter electromotive force of induction would also have a sine form, and would therefore be proportional to the effective pressure determined by

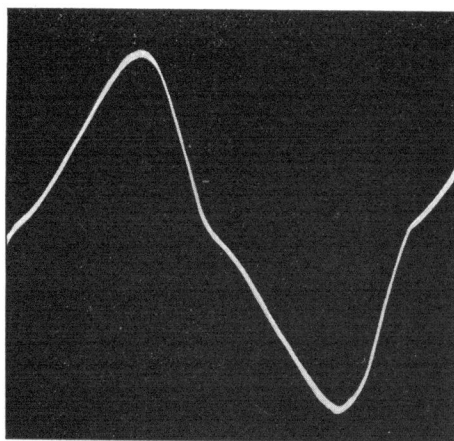


FIG. 2.

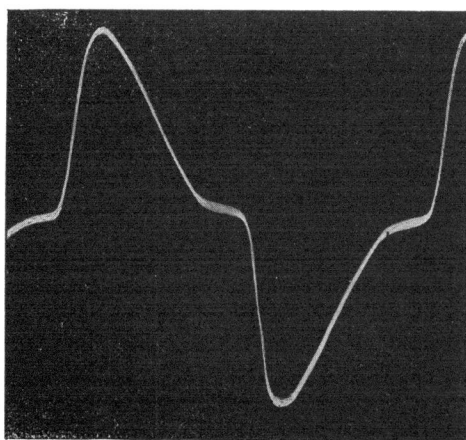


FIG. 3.

the voltmeter. This, however, is practically impossible, as there is always more or less impedance in the primary circuit external to the magnetizing coil; namely, that of the instruments in the circuit, as well as that of the transformer for stepping down the generator pressure to a sufficiently low value to apply to

the sample under test. In fact, even when the utmost precautions are taken to keep the impedance of the circuit exterior to the magnetizing coil as small as possible, this external impedance may be the controlling factor in the circuit, particularly for high values of the flux-density, and corresponding low values of the permeability. As this external impedance is practically constant under such conditions, the current in the magnetizing coil may have very nearly a sine form, and consequently produce a much distorted wave of induced electromotive force in the sample.

The accompanying oscillograph records give the pressure-waves under various conditions induced in a secondary winding on a sample of iron weighing about 6 lb. The sample was built up of punchings in the form of a hollow square 5 in. outside, 3 in. inside, each sheet 0.14-in. thick. The magnetizing coil had 20 turns. The total resistance of the circuit between generator and sample was about 0.75 ohm, of which 0.3 ohm was in the instruments in the circuit and 0.45 in the step-down transformer (one-half a kilowatt, 20 to 1 ratio). The magnetizing coil itself had practically no resistance. The source of pressure was a small 110-volt motor-generator, giving a sine-wave electromotive force.

Curve 1 is for 25 cycles and a maximum flux-density of 6,850 lines of induction per square centimeter.

Curve 2 is for 25 cycles and a maximum flux-density of 11,300

Curve 3 is for 25 cycles and a maximum flux-density of 12,800.

A sine-wave of pressure at the terminals of the magnetizing coil would have given flux-densities of 7,000, 12,000 and 15,000 lines respectively.

The following table gives for another sample the flux-densities calculated on the basis of a sine wave electromotive force and the true flux-densities determined from the average values of the electromotive force wave:

| B<br>Calculated on basis<br>of sine wave. | True B     |            | Ratio of True B to Calculated B. |            |
|-------------------------------------------|------------|------------|----------------------------------|------------|
|                                           | 60 Cycles. | 25 Cycles. | 60 Cycles.                       | 25 Cycles. |
| 4,000                                     | 4,000      | 4,000      | 1.00                             | 1.00       |
| 7,000                                     | 6,800      | 6,500      | 0.97                             | 0.93       |
| 10,000                                    | 9,400      | 9,100      | 0.94                             | 0.91       |
| 12,000                                    | 11,000     | 10,600     | 0.92                             | 0.88       |
| 15,000                                    |            | 12,200     |                                  | 0.81       |
| 17,500                                    |            | 13,200     |                                  | 0.76       |
| 20,000                                    |            | 14,100     |                                  | 0.71       |

**A. Henry Pikler** (by letter): Professor Bedell's paper gives the impression that the hysteresis loop is the only cause of the distortion of alternating-current wave-forms. This is not so. The

distortion is primarily due to the change in permeability during one cycle of magnetism; the hysteresis is only accidental.

Mr. Steinmetz indicates how the induced electromotive force wave form will become distorted in a transformer by magnetizing it with a sine wave current. To sustain this contention, I refer to my oscillograms taken in June, 1903,\* There it is shown that a constant transformer terminal voltage can be maintained by various forms of magnetizing-current waves which, as they become more and more sinusoidal will make the transformer electromotive force wave take the shape of the previously distorted-current wave.

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\**Electrical World and Engineer*, 1903, Vol. XLII, No. 6, p, 218.